AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

NOTE ON PROBLEM 26, AVERAGE AND PROBABILITY.

BY G. B. M. ZERR.

In reply to Dr. Martin, for whom I have the utmost respect, I have the following remarks to make. The problem that gives the result $\frac{1}{6}a^2$ is different from the problem that gives the result $\frac{a^2}{2\pi}$. In the former the right angle remains fixed and does not lie on a circle as Dr. Martin states. The problem is as follows: Find the average area of all triangles formed by a straight line of constant length a sliding so that its extremities constantly touch two fixed straight lines at right angles to one another. In the problem under consideration the hypotenuse a is fixed and the right angle moves on the semi-circumference. In the first case the average length of one leg is $\int_0^a x dx \Big/ \int_0^a dx = \frac{1}{2}a$. In the second case the average length is $\int_0^a x ds \Big/ \int_0^a ds = a/\pi$. In the first case the average area of all the triangles is $\int_0^a \frac{1}{2}x\sqrt{a^2-x^2}dx \Big/ \int_0^a dx = \frac{1}{6}a^2$. In the second case the average area is $\int_0^a \frac{1}{2}x\sqrt{a^2-x^2}ds \Big/ \int_0^a ds = \frac{a^2}{2\pi}$, where ds represents an element of arc. It is plainly evident that in the result $\frac{1}{6}a^2$ the leg does not and cannot change its direction or its average length would not be $\frac{1}{2}a$. In the second case it is constantly changing its direction and the right angle is moving on a semicircumference. The problem calls for a given hypotenuse and not one that is constantly changing its direction; hence the result $\frac{a^2}{2\pi}$ is the correct result.

DR. MARTIN'S RESULT IS NOT CORRECT.

F. P. MATZ.

Cause the problem to read: "Find the average area of all right-angled triangles having a given hypotenuse, *if an arm of the triangle vary uniformly;*" then Dr. Martin's result, $\frac{1}{6}h^2$, is perfectly correct.

Strip the problem of this italicised condition; that is, make the problem read as originally proposed; then the number of possible right-angled triangles is proportional to the *length* of the semicircumference of which the given hypotenuse is the diameter. This is *the correct* plan of solution. By *adhering* to *this* plan of solution, the correct result, $h^2/2\pi$, is obtained, regardless as to choice of independent variable.

Dr. Martin's result, $\frac{1}{6}h^2$, is *too great;* for he, by making the number of possible right-angled triangles "proportional to the given hypotenuse," *ignores*